Physics 8B  

MIDTERM 2  

Spring 2002 (Section 1)  

1. (20 points) A solenoid contains 300 turns/cm and has a radius of 2.0 cm. It carries a time-varying current given by \( i(t) = 0.10t^2 + 0.50 \), measured in amps. At \( t = 2.0 \) s, find the magnitude of the electric field induced at a distance of (a) 1.0 cm and (b) 3.0 cm from the solenoid axis. 

Note: The magnetic permeability constant \( \mu_0 \) is given by \( 4\pi \times 10^{-7} \) T·m/A. 

2. (15 points) A ray of light traveling through water enters a piece of glass, as shown below. It then strikes a glass-air interface at the minimum angle necessary for total internal reflection. Find the angle that the original ray makes with the surface of the glass. Take the indices of refraction of glass, water, and air to be 1.50, 1.33, and 1.00, respectively. 

3. (15 points) An object is placed 15.0 cm in front of a mirror. The resulting image is upright, and has a magnification of 0.60. Find (a) the location of the image (i.e., the distance from the mirror, and whether it is in front of or behind the mirror), and (b) the focal length of the mirror. (c) Is it concave or convex? (d) Draw a ray diagram. (You only need to locate the image with two rays, but you may use more if that helps you.)
Midterm 2 Solutions

1. \[ \oint E \cdot d\vec{S} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(\Phi_A) = -A\frac{d\Phi}{dt} \]

So the magnitude of \( E \) is given by

\[ E \cdot 2\pi r = A\frac{d\Phi}{dt} \tag{2} \]

But \( B = \frac{\mu_0 i}{2\pi r} \), so

\[ E = \frac{A}{2\pi r} \cdot \frac{\mu_0 i}{2\pi r} \frac{di}{dt} \]

\[ \frac{di}{dt} = \frac{d}{dt} (0.10^2 + 0.50) = 0.20 \Rightarrow (0.20)(2.0) = 0.40 \Rightarrow A \tag{3} \]

(a) \( A = \pi r^2 \), so

\[ E = \frac{\pi r^2}{2\pi r} \cdot \frac{\mu_0 i}{2\pi r} \frac{di}{dt} = \frac{r^2}{2} \frac{\mu_0 i}{2\pi r} \frac{di}{dt} \]

\[ = \frac{(0.011)(4\pi \times 10^{-7})(30000)(0.40)}{2} \]

\[ = 7.5 \times 10^{-5} \text{ V/m (or N/m²)} \tag{3} \]

(b) \( A = \pi R^2 \), \( R = 2.0 \text{ cm, so} \)

\[ E = \frac{\pi R^2}{2\pi r} \cdot \frac{\mu_0 i}{2\pi r} \frac{di}{dt} = \frac{R^2}{2r} \frac{\mu_0 i}{2\pi r} \frac{di}{dt} \]

\[ = \frac{(0.021^2)(4\pi \times 10^{-7})(30000)(0.40)}{2(0.03)} \]

\[ = 1.0 \times 10^{-4} \text{ V/m (or N/m²)} \tag{3} \]
The critical angle is given by

\[ n_{\text{glass}} \sin \theta_c = n_{\text{air}} \]

\[ \sin \theta_c = \frac{n_{\text{air}}}{n_{\text{glass}}} = \frac{1.00}{1.50} \]

\[ \theta_c = 41.8^\circ \quad (5) \]

So the angle of refraction at the water-glass interface is \( 90^\circ - 41.8^\circ = 48.2^\circ \) \( (3) \)

So the angle of incidence is given by

\[ n_{\text{water}} \sin \theta_1 = n_{\text{glass}} \sin \theta_c \]

\[ 1.33 \sin \theta_1 = 1.50 \sin 48.2^\circ \]

\[ \theta_1 = 57.2^\circ \quad (5) \]

So the angle it makes with the surface is \( 90^\circ - 57.2^\circ = 32.8^\circ \) \( (2) \)
3. (a) \( m = \frac{1}{p}, \quad m = +0.60 \)

\[ i = -m \cdot p = -(16.60)(15.0) \]

\[ = -9.0 \text{ cm} \hspace{1cm} (4) \]

The image is 9.0 cm behind the mirror.

(b) \[ \frac{1}{f} = \frac{1}{p} + \frac{1}{i} = \frac{1}{15.0} + \frac{1}{-9.0} \]

\[ f = -22.5 \text{ cm} \]

(c) Convex, since \( f > 0 \) \hspace{1cm} (2)

(d)