University of California at Berkeley
Department of Physics
Physics 8A, Spring 2003

Midterm 1
March 5, 2003

You will be given 180 minutes to work this exam. No books are allowed, but you may use handwritten formulae sheet no larger than one side of an 8 1/2 by 11 sheet of paper. Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, and take particular care to explain what you are doing. Please use the symbols described in the problems, tell us why you’re writing any new equations, and label any drawings that you make. Write your answers directly on the exam, and if you have to use the back of a sheet make sure to put a note on the front. Do not use a blue book or scratch paper.

\[ v = \frac{dx}{dt} \quad a = \frac{dv}{dt} \quad x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \quad \sum \vec{F} = m \vec{a} \quad F_e = \frac{mv^2}{r} \]

\[ F_k = \mu_k N \quad W = Fx \quad P = \frac{dW}{dt} \quad K = \frac{1}{2} m v^2 \quad U = mgh \quad D = \frac{1}{2} C \rho A v^2 \]

\[ \Delta K = K_f - K_i \quad \Delta U = U_f - U_i \quad W = \Delta U + \Delta K + \Delta E_{ext} + \Delta E_{int} \]

\[ v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \quad v_{2f} = \frac{2m_2}{m_1 + m_2} v_{1i} + \frac{m_3 - m_1}{m_1 + m_2} v_{2i} \]

NAME: ______________________

SID NUMBER: ______________________

DISCUSSION SECTION NUMBER: ______________

DISCUSSION SECTION DATE/TIME: ______________

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

106
1) (20 points) Sliding block
A block slides down a track that starts with a quarter of a circle, and ends with a flat and level part. At the end of the track, it hits and sticks to a stop. Note that the ramp is free to move left and right. You can ignore friction in this problem. The mass of the block is $m_1$, and the mass of the ramp is $m_2$. The radius of the circular part of the ramp is $R$, and the length of the straight part is $L$. If you use conservation laws to solve any part of this problem, make it really clear which ones you’re using, and how you’re applying them.

(a) After the block has stuck to the ramp, are the combined ramp & block moving to the left, to the right or stationary?

(b) Between the time the block started moving and the time that the block hit the stop, did the center of mass of the two objects move up or down? Did it move right or left?

(c) What is the position of the block when it hits the stop? Note that the diagram as an $x$=0 point defined, please use that.

\[
\begin{align*}
\text{(a)} & \quad \text{Total initial momentum for the system } = 0 \\
& \quad \Sigma \vec{p}_x = 0 \\
& \quad \text{After the block has stuck to the ramp, once again its momentum is zero and hence ramp momentum should be zero as well. Therefore they'll be "stationary".}
\end{align*}
\]

\[
\begin{align*}
\text{(b)} & \quad \text{According to the law of conservation of center of mass, since there is no external force in the "x" direction, the x-position of center of mass remains constant but along y there is gravity and since the smaller mass moves down the center of mass comes down as well.}
\end{align*}
\]

\[
\begin{align*}
\text{(c)} & \quad \frac{m_1v_1 - m_2v_2}{2} = 0 \\
& \quad \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = m_1gR \\
& \quad \Rightarrow \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\frac{m_2^2}{m_2}v_1^2 = m_1gR \\
& \quad \Rightarrow v_1^2 + \frac{m_2}{2m_2}v_1^2 = gR
\end{align*}
\]
\[ x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1}{m_1 + m_2} + \frac{m_2 x_2}{m_1 + m_2} \]

Let the ramp move to left by \( x \),

\[ x_{CM} = \frac{m_1 (R + L - x) + m_2 (x_2 - x)}{m_1 + m_2} \]

\[ = \frac{m_1 (R + L - x) + m_2 x + m_2 x_2}{m_1 + m_2} \]

\[ = \frac{m_2 x_2}{m_1 + m_2} \]

\[ m_1 (R + L - x) = m_2 x \]

\[ \Rightarrow m_1 (R + L) = (m_1 + m_2) x \]

\[ \Rightarrow x = \frac{m_1}{m_1 + m_2} (R + L) \]

\[ R + L - x = \frac{m_1}{m_1} x \quad \text{is the position of the block.} \]

\[ = \frac{m_2}{m_1} \cdot \frac{m_1}{m_1 + m_2} (R + L) \]

\[ = \frac{m_2}{m_1 + m_2} (R + L) \]
2) (20 points) Two-part projectile motion
A ball is thrown up onto a flat horizontal roof. The ball lands on the roof at the highest point of its path. It then rolls across the roof, and falls off on the other side. (See diagram)
Ignore friction, and what we've learned this week about rotational energy. Call the initial speed $v_0$ and angle $\theta$, and use L for the length of the building.

Find the time between when the ball is thrown, and when it lands on the far side, in terms of the variables above.

\[
\text{Total time} = t_1 + \frac{L}{v_0 \cos \theta} + t_2
\]

\[
= \frac{v_0 \sin \theta}{g} + \frac{L}{v_0 \cos \theta} + \frac{v_0 \sin \theta}{g}
\]

\[
= \frac{2v_0 \sin \theta}{g} + \frac{L}{v_0 \cos \theta}
\]
3) (20 points) Two blocks
Two blocks are connected by a string as shown. Friction cannot be ignored. Use $m_1$ and $m_2$ for the masses of the two blocks, and $\mu_s$ and $\mu_k$ for the coefficients of static and kinetic friction. You can assume that $m_1$ is larger than $m_2$.

Using just the information given, determine whether the blocks are sliding or not. (We're looking for an answer like "if this is bigger than that, they're sliding")

What is the tension in the string in that case?

\[ T - m_2 g \cos \theta \geq \mu_s N \quad \text{(1)} \]
\[ m_1 g - T \geq 0 \quad \text{(2)} \]
\[ m_1 g \geq T \quad T = \mu_s m_2 g \sin \theta + m_2 g \cos \theta \]
\[ \frac{m_1 g}{\mu_s (\sin \theta + \cos \theta)} \quad \text{they will slide} \]

Then:
\[ T - m_2 g \cos \theta - \mu_k N = m_2 a \]
\[ m_1 g - T = m_1 a \]
\[ m_1 T - m_1 m_2 g \cos \theta - \mu_k m_1 m_2 g \sin \theta = m_1 m_2 g - m_2 T \]
\[ T = \frac{m_1 m_2 g (\cos \theta + \mu_k \sin \theta)}{(m_1 + m_2)} \]
4) (20 points) Train into buffer
A 50 ton (50x10³ kg) locomotive is pulling into a station when the engineer realizes he is going too fast (v=10 m/sec). He locks the brakes, so the train starts to slide, when it's a distance d=80m from the end of the track. The coefficient of friction is 0.1 while the train is sliding. At the end of the track is a spring, which is strong enough bring a train to a stop. It has a spring constant k = 3x10⁴.

Please do each part of this problem symbolically, only plugging in numbers at the end.

a) With the numbers as given above, does the train hit the spring at the end of the track?

b) If the train were to hit the spring while moving at 1m/sec, how far would the spring compress?

(a) Locomotive mass \( m \), speed \( v \), distance \( d \), \( \mu \) - coefficient of friction

\[
\frac{1}{2}mv^2 - \mu mgd = \text{Remaining energy to compress the spring,}
\]

\[
\frac{1}{2} \times 50 \times 10^3 \times 100 - 0.1 \times 50 \times 10^3 \times 10 \times 80
\]

\[
= 2,500,000 - 400,000 = 2,100,000 \text{ which is absurd and hence it implies the train comes to a stop before hitting the spring.}
\]

(b)

\[
\frac{1}{2}mv^2 = \frac{1}{2}kx^2
\]

\[
\Rightarrow x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{50 \times 10^3 \times 100}{3 \times 10^4}} = \sqrt{\frac{5000}{3}} = 1.29 \text{ m}
\]
5) (20 points) Falling spheres

Two spherical balls are dropped from a very large height. One has a radius of 3 cm, and the other has a radius of 12 cm. The mass of both balls is 0.5 kg.

a) After they reach terminal velocity, one is moving much faster than the other. Why?

b) Which ball will have greater kinetic energy when it hits the ground? The larger one, or the smaller one? Why?

c) Find the ratio of their kinetic energies when they hit the ground.

\[ (A) \quad \text{Since the larger ball has greater air drag it is moving slower than the smaller ball.} \]  
\[ \text{(greater area of exposure)} \]

\[ (B) \quad \text{The smaller one as it is moving faster due to less drag} \]

\[ (C) \quad \frac{m_1 g}{m_2 g} = \frac{C_D}{C_D} \frac{\frac{1}{2} \rho g \pi A_1 V_1^2}{\frac{1}{2} \rho g \pi A_2 V_2^2} = \left( \frac{m_1}{m_2} \right)^2 \left( \frac{A_2}{A_1} \right) \]

\[ \frac{m_1 g}{m_2 g} = \frac{\frac{1}{2} m_1 \rho \frac{1}{2} \pi A_1}{\frac{1}{2} m_2 \rho \frac{1}{2} \pi A_2} = \frac{\frac{1}{2} \rho g \pi A_1}{\frac{1}{2} \rho g \pi A_2} = \frac{m_1^2}{m_2^2} \frac{A_1}{A_2} \]

\[ CD = \text{Drag Coefficient} \]
\[ A_i = \text{Projected area of spheres} \]
\[ V_i = \text{Terminal velocities} \]
\[ \left( \frac{m_1}{m_2} \right)^2 \left( \frac{A_2}{A_1} \right) = \left( \frac{12}{3} \right)^2 = 16 \text{-times} \]