You will be given 80 minutes to work this exam. No books are allowed, but you may use a handwritten note sheet no larger than an 8 1/2 by 11 sheet of paper. Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, and take particular care to explain what you are doing. Please use the symbols described in the problems, tell us why you're writing any new equations, and label any drawings that you make. Write your answers directly on the exam, and if you have to use the back of a sheet make sure to put a note on the front. Do not use a blue book or scratch paper.

\[ \begin{align*}
    v &= \frac{dx}{dt} \\
    a &= \frac{dv}{dt} \\
    x(t) &= x_0 + v_0 t + \frac{1}{2} a t^2 \\
    \sum F &= m a \\
    F_c &= \frac{mv^2}{r} \\
    F_k &= \mu_k N \\
    W &= F_x \\
    P &= \frac{dW}{dt} \\
    K &= \frac{1}{2} m v^2 \\
    U &= mgh \\
    \Delta K &= K_f - K_i \\
    \Delta U &= U_f - U_i \\
    W &= \Delta U + \Delta K + \Delta E_{\text{el}} + \Delta E_{\text{int}}
\end{align*} \]
1) (20 points) Roller coaster motor (power)

The motor on a roller coaster pulls a car of mass $M$ to the top of the first hill at a constant velocity $V$. The hill is $D$ meters long, and is at an angle $\theta$ (see sketch).

a) During the lift, what is the power provided by the motor?

b) During the lift, what is the force provided by the motor?

\[
\text{Power} = \frac{\Delta W}{\Delta t} = F \cdot V \cos \theta = \frac{dE}{dt}
\]

Work = $F \cdot d$

\[
\begin{align*}
\text{Bottom} & \quad v=0 \rightarrow KE=0 \\
& \quad u=0 \\
\text{Work done by motor} = E_f - E_i = \frac{1}{2}mv^2 + mgD\sin\theta
\end{align*}
\]

\[
\text{Power} = \frac{d}{dt} \left( \frac{1}{2}mv^2 + mgD\sin\theta \right) = \frac{1}{2}mV^2 + mgD\sin\theta = \frac{1}{2}mV^2 + mgD\sin\theta
\]

\[
\sum F = ma
\]

\[
\text{Force} - mg\sin\theta = ma = 0
\]

\[
\text{Force} = mg\sin\theta
\]

\[
\cos \phi = 1.
\]

Power and velocity are in the same direction, so

\[
P = F \cdot V = FV\cos\theta = 0
\]

\[
P = FV
\]

\[
-mgV\sin\theta \quad \phi < 0
\]
2) (20 points) Spring ping-pong

Two springs with different spring constants $k_1$ and $k_2$ are attached to walls a distance $D$ apart, with a frictionless surface between them (see illustration). A body of mass $M$ is placed in front of the left one, and the spring is compressed to a distance $d$. At time $t=0$, the body is released and starts to move.

a) Draw a plot of the momentum of the block as a function of time. It should cover a long enough time to show all of the various things that happen to the block as it moves back and forth.

b) When it hits the spring on the right, how far does that spring compress?
(20 points) Projectile motion

A projectile of mass $m$ is fired upwards from level ground at an angle $\theta > 45$ degrees with a velocity $v_0$. (See sketch; note that a specific coordinate system is defined)

a) What is the smallest speed it has during the flight?

b) The horizontal component of the velocity is equal to the vertical component of the velocity at one point. How long does it take the projectile to reach that point?

Correct answer (a):

- $v_x = v_0 \cos \theta$
- $v_y = v_0 \sin \theta - gt$
- At top of arch, $v_y = 0$
- Smallest speed in $v_x$ at top of arch

Correct answer (b):

- $v_{x0} = v_x = v_0 \cos \theta$
- $v_{y0} = v_0 \sin \theta$
- $v_y(t) = v_{y0} \sin \theta - gt$
- $v_{x0} \sin \theta - gt = v_0 \cos \theta$

$t = \frac{v_0}{g} \left( \sin \theta - \cos \theta \right)$
4) (20 points) Frictional braking

An object of mass \( M_1 \) is moving to the left with speed \( s_1 \) past the point \( x = 0 \) when it suddenly collides with an object of mass \( M_2 \), moving at speed \( s_2 \). (See illustration). Note that \( s_1 \) and \( s_2 \) are speeds, hence are positive numbers, but one object is moving to the left and one to the right. After the collision they stick together.

a) In terms of \( M_1, M_2, s_1 \), and \( s_2 \), determine whether they will move to the left or the right after the collision.

b) After the collision, they move together with a coefficient of kinetic friction \( \mu_k \) between the blocks and the surface. How far do they move before stopping?

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\[
\Delta P = 0 \\
P_0 = P_f \\
(m_1 s_1) + (m_2 (-s_2)) = (m_1 + m_2) s_f
\]

\[
s_f = \frac{m_1 s_1 - m_2 s_2}{m_1 + m_2}
\]

If \( m_1 s_1 > m_2 s_2 \), then \( s_f > 0 \) and both cars move to the right after colliding.

If \( m_1 s_1 < m_2 s_2 \), then \( s_f < 0 \) and both cars move to the left after colliding.

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\[
\text{Conservation of energy} \\
KE + W_f = 0 \\
\frac{1}{2} (m_1 + m_2) s_f^2 + F_f \cdot d = 0 \rightarrow \frac{1}{2} (m_1 + m_2) s_f^2 = -[\mu_k (m_1 + m_2) g] \cdot d \\
\frac{1}{2} (m_1 + m_2) s_f^2 = \mu_k (m_1 + m_2) g \cdot d
\]

\[
d = \frac{s_f^2}{2 (\mu_k g)}
\]

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\[
N = (m_1 + m_2) g \\
F_f = \mu_k N = \mu_k (m_1 + m_2) g = (m_1 + m_2) \alpha
\]
5) (10 points) The Scale

A professor is standing on a "scale" in an elevator. The scale measures the force exerted on the floor by the professor. It reads 1000 Newtons when the elevator is stationary (assume g=10 m/s²). What does it read if the elevator is accelerating upward at 20 m/s²?

\[ \Sigma F_y = ma \]
\[ N - mg = ma \]
\[ N = M(g+a) \]

If \( a = 0 \),
\[ 1000 = m(10+0) \]
\[ m = 100 \text{ kg} \]

If \( a = 20m/s^2 \),
\[ N = M(g+a) \]
\[ = 10(10+20) \]
\[ = 3000 \text{ Newtons} \]
6) (2 points each) Miscellaneous

6.1) Of the following situations, which one is impossible?
   a) a body having velocity east and acceleration east
   b) a body having velocity east and acceleration west
   c) a body having zero velocity and non-zero acceleration
   d) a body having constant acceleration and variable velocity
   e) a body having constant velocity and variable acceleration

6.2) A particle moves at a constant speed in a circular path. The instantaneous velocity and instantaneous acceleration vectors are:
   a) both tangent to the circular path
   b) both perpendicular to the circular path
   c) perpendicular to each other
   d) opposite to each other
   e) none of the above

6.3) A "newton" is the force
   a) of gravity on a 1 kg body
   b) of gravity on a 1 gram body
   c) that gives a 1 gram body an acceleration of 1 cm/s²
   d) that gives a 1 kilogram body an acceleration of 1 m/s²
   e) that gives a 1 kilogram body an acceleration of 9.8 m/s²

6.4) In a tug-of-war, two students each pull on the rope with 100 Newton forces, in opposite directions. The tension in the rope is
   a) zero
   b) 50 Newton
   c) 100 Newton
   d) 141 Newton
   e) 200 Newton

6.5) A heavy steel ball B is suspended by a cord from a block of wood W. The entire system is dropped through the air. Neglecting air resistance, the tension in the cord while falling is:
   a) zero
   b) the difference in the masses of B and W
   c) the difference in the weights of B and W
   d) the weight of B
   e) none of these